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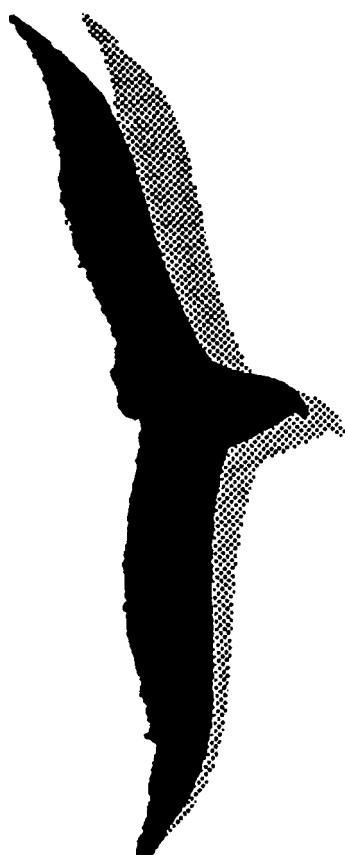
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Labour flows in a simulation model of the firm

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LABOUR FLOWS IN A SIMULATION MODEL OF THE FIRM

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and

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Summary

A hierarchical model is calibrated and used to illustrate labour market flows within a **firm**. The model establishes a link between the models of the firm **from** the literature on industrial organisation and the description of labour market dynamics in the flow approach to labour markets. It describes the decision of the personnel management of the firm whether to fire workers, and/or whether to hire workers from the internal or external labour market. The decision is based on firing costs, hiring costs, training costs and the availability of qualified workers within and outside the firm. The firm tries to adjust the size and composition of its personnel as much as possible to the optimal size and hierarchical structure which follows from maximisation of the net revenues in each period. Data in this maximisation process of the **firm** are productivity, wage differentials, economies of scale from cooperation, foregone production due to supervising and training, and average hiring and firing costs. Yet, usually immediate adjustment to the optimal composition of the personnel is not possible due to unanticipated quits and due to lack of qualified workers. In that case vacancies may remain open for a prolonged time. The parameters of the model are calibrated in order to mimic a representative **firm** of average size, which has external and internal labour flows in accordance with data found in the (scanty) empirical literature. Heterogeneity amongst firms is introduced by means of changes in the major parameters which dictate the hierarchical structure and optimal size of the firm.

key words:

hierarchical structure of a firm, internal labour market, hiring and firing costs, job mobility, costs and benefits of training

1. Introduction

Analysis of large longitudinal micro data sets at the level of the firm has recently provided much insight into labour market dynamics. The seminal work by Leonard (1994), Dunne *et al.* (1989) and Davis & Haltiwanger (1990, 1992) has shown that gross job and labour flows are much larger than net changes in employment. Many replication studies and extensions of the work of Davis & Haltiwanger on job creation and job destruction indicate a considerable convergence of international evidence with respect to various characteristics of labour market dynamics (see e.g. Davis, Haltiwanger & Schuh, 1996, and the references therein). One major finding is that there is much heterogeneity amongst firms and establishments, and that the largest part of job turnover (job creation + job destruction) takes place within the same regions and branches of industry. It implies that job creation and job destruction is much more driven by idiosyncratic, firm specific shocks, than by demand and supply shocks at the macro level.

In recent years also much effort has been spent on small theoretical models which **may** explain these stylised facts of labour market dynamics. A major background is equilibrium search theory, which explains how dynamic unemployment equilibria are the result of transaction costs in the search process between employers and employees. Heterogeneities at the labour market are a main reason for these transaction costs, which make the matching process between job seekers and employers who have vacant positions, time consuming. Because of the transaction costs and the possibility of mismatches, there is a surplus value of a successful match, which is to be shared between the worker and the employer.

However, these equilibrium search models and the related labour demand models, in which transaction costs play a major role in explaining unemployment equilibria and sluggish adjustment dynamics, are based on elegant but highly abstract assumptions on labour market behaviour of firms and workers. For instance, the idea that the employer and the successful applicant bargain in their wage negotiations over the distribution of the surplus value of the match, looks somewhat far-off from the way wages of new workers are set according to fixed salary scales in the real world. Hence, the abstract models of equilibrium search do not provide insight into the various mechanisms that govern labour market flows at the level of the firm (and do not purport to do so). Moreover, these models do not reckon with labour market flows within the firm, i.e. with internal job mobility. These internal labour flows are, by the way, often neglected in the empirical analysis of labour market dynamics along the lines of Davis & Haltiwanger (see Hamermesh *et al.*, 1996, for a counter example).

We want to unveil the labour mobility within the ‘black box’ of the firm. Obviously, internal and external labour flows are closely connected. Some of the fundamental questions about the relation between jobs and employees can be answered only at the level of the firm. When does a firm open a vacancy, and what determines the design of the vacancy? Are job requirements only based on the needs of the production or do personnel managers adjust the job to fit the available employees? When a firm has a vacancy • either

through creation of a new job, through a quit or through an internal move • the costs of hiring an external candidate are to be balanced against the costs of moving an internal candidate. Factors like the career prospect within the firm, and the employability in other firms are important in the study of labour mobility and the process of job creation and destruction.

In this paper we focus on the construction of a model that formalises the decisions of personnel managers. The main feature of the model is that explicitly describes the choice between external mobility and labour mobility within the firm. We design a stylised hierarchical simulation model of the firm that combines elements of dynamic search theory with elements of internal labour market theory. In this model we are able to follow the career of each employee in the firm, including his entry and exit. Hereto we extend Williamson's (1967) model of the relationship between firm size and hierarchical control with a number of features which allow us to generate internal and external labour market flows using the simulation model. These features are the inclusion of

1. the stochastics of quits,
2. hiring, firing, training and supervision costs,
3. the building up of skills of incumbent workers through learning by doing and training,
4. defining the level of skills required at each hierarchical layer, and
5. the skill distribution of job seekers available at the external labour market.

Obviously such a simulation model is not a neat and elegant model which can be analytically solved, like the models of the equilibrium search theory. Our model can only be solved numerically. It has to be calibrated with respect to numerous functional forms and parameter values. In this calibration procedure we take account of unexpected idiosyncratic shocks which hit the firm through the specification of the stochastics of quits and the skill distribution of job seekers which mimic the type of shocks considered in the models of the flow approach to the labour market. Heterogeneity amongst firms is included in the model through changes in the parameter values and functional forms of e.g. the production function, the skill requirements in the various hierarchical layers, the span of control of supervisors (defining the flatness of the organisation), the hiring, firing and training costs etc.. As far as we know equilibrium search models of the labour market do not take account of these kinds of heterogeneity between firms.

The model is calibrated in a simulation exercise with 100 replications so that its dynamic base line simulation reproduces some stylised facts from empirical studies of the industrial organisation on the size distribution of firms. Moreover, in the calibration the average sizes of the internal and external labour market flows should be in line with what seems plausible and with the scant empirical evidence in this respect.

The next section reviews some of the stylised facts and discusses some relevant assumptions and implications of equilibrium search theory and internal labour market theory. Section 3 presents Williamson's original hierarchical model of the firm and discusses the necessary extensions to the model inspired by search theory so that the model generates labour flows at the level of the firm. Section 4 gives the main features and characteristics of the simulation model, whereas section 5 discusses the central projection

of the calibrated model. Section 6 illustrates the sensitivity of the working of the model in case of specification changes and changes in parameter values, in order to gain insight in the sources of observed heterogeneity amongst firms. Section 7 concludes and gives some suggestions for further research in which the model of this paper can be a tool.

2. Stylised facts and implications from internal labour market theory and the flow approach to labour markets

The conventional neo-classical labour market theory, with homogeneous labour supply, elastic labour conditions, perfect information, perfect competition and rational behaviour by representative agents does not seem to be much in line with the observed behaviour of employers and employees. There is much less mobility between jobs than implied by the neo-classical theory. Tenure is often long, and mobility low. To capture this phenomenon, the concept of dual or segmented labour markets, and more specific the concept of the internal labour market is developed.

Discussions about the theory of the internal labour market often use the definition of Doeringer & Piore (1971) as the starting point. They **define** the internal labour market as *“an administrative unit, such as a manufacturing plant, within which the pricing and allocation of labor is governed by a set of administrative rules and procedures”* (p. 1, 2). It is clear that every internal labour market is linked to an external labour market, where the conventional (neo-classical) theory of pricing, allocating and training decisions governed directly by economic variables is assumed to be applicable. Optimising behaviour, which is a basic part of the neo-classical theory, plays a minor role in internal labour markets, but it is too simple to conclude that the ILM takes an anti efficiency approach. The occurrence of ILMs can be efficient when the assumptions underlying the neo-classical theories are relieved. For example, the existence of turnover costs can make it profitable to have more or less fixed job ladders instead of competition between all workers. Adjustment costs, i.e. costs involved with hiring, **firing** and training of employees, will play an important role when workers are heterogeneous and when **firm-specific** skills are necessary. Both the employee and the employer can benefit **from** the existence of an internal labour market: the employee has more job security while the employer is guaranteed of good workers at relatively low costs.

Of course, as the internal and external markets are interconnected, there must be some job classifications at which entry to and exit **from** internal markets is possible. In general, the ILM-concept gives certain rights and privileges to employees already incorporated in the firm, e.g. career paths, and to a certain extent they are protected from competition with persons who try to enter the firm. Doeringer & Piore make a difference between closed and open ILMs. The closed ILM has only one entry level, where the open ILM is characterised by the existence of many entrance ports. Governance by economic variables like the unemployment rate, wages paid by competitors and so on, will be important only at these entry levels. Wages in ILMs are related to jobs and not to the productivity of individual workers, as in neo-classical theory. Only the wages for jobs at the entry level

must have a competitive level. Promotions are not solely based on ability, other factors like seniority **often** play a major role.

It is questionable whether the definition of Doeringer & Piore, although it is generally used and accepted, is useful in practical research. An option for practical research is to follow the flows of employees through the firm, as it is difficult to discover the underlying rules, procedures, habits etc., and their effects on the **final** decision on internal and external hiring. However, due to lack of data, research on internal flows is rare and often derived from information on external flows. See, e.g., the study on the characteristics of **ILMs** by Van Bergeijk & De Grip (1986), which uses data **from** the Annual Social Reports of 19 large Dutch firms. They observe that on average about one-third of the employees have job tenure of over ten years, while about two-third of the personnel has more than five years of tenure. Between 1974 and 1983, tenure in these firms has increased **from** 8 to just over 10 years on average. In the early **80's**, almost all firms have (external) mobility rates of less than 10%. Information on the occurrence of flows through the organisation is not available. An exceptional case where information on internal flows is available is Hamermesh *et al.*, (1996). This study estimates the sizes of various internal and external worker flows in a large sample of Dutch firms. Flows of workers within firms appear to be small compared to flows into and out of **firms**

An elaborated case study is done by Baker *et al.* (1993,1994), who identified the hierarchical structure of a medium-sized U.S. firm in a services industry, based on the personnel files over the years 1969-1988, and by observing the career paths of the (management) employees. A large number of employees have lengthy careers in the firm, and these careers are characterised by movements through numerous jobs, which is an indication for the existence of an **ILM** in the **firm**. However, there is no evidence on the existence of specific ports of entry and exit. Both occur in all jobs and at all levels. There is little difference in quality between personnel hired externally and employees who are hired internally; firm-specific human capital does not seem to be important. Career performance does not differ between internal and external hires, tenure does not result in better career attainment, and “fast tracks” do exist. These are all indications that promotions are based on performance and qualities of employees, and not so much on seniority. There is strong evidence for the importance of job levels as determinants of the wages, but wage jumps at promotions are much smaller than differences across levels and within levels. In general, administrative rules in wage determination seem to be important so that the neo-classical assumption that wages reflect marginal productivity seems to be falsified.

In our simulation model, we will concentrate on the mobility of employees in the **firm**. We will try to implement a number of empirical facts discovered by Baker *et al.* and, therefore, keep some distance from the **ILM** theory of Doeringer & Piore. Decisions about hiring are taken to **find** an efficient mode of production. In this decision we assume that the firm weights the benefits and costs of internal mobility against external hiring; we abstract from promotions-guided-by-procedures. A case study of a large Dutch manufacturer by Van

Veen (1997) supports our efficiency-approach. Van Veen shows that there exists a system of formal rules in the firm, but that it is adjusted regularly to match changes in the environment of the **firm** and rationalise the mobility of employees. In the calibration procedure we try to select the specifications and parameter values of the model in such a way that it reproduces empirical evidence on internal and external labour market flows found by Hamermesh *et al.* and evidence on personnel management and hierarchical structure by Baker et al. Therefore we are to endogenise the hierarchical structure, tenure and consequently internal and external labour flows in our simulation model of the **firm** and derive these characteristics from optimising behaviour.

3. Hierarchical models of the firm

As a starting point for our simulation model, we use the hierarchical model of Williamson (1967), a simple model that gives a clear insight in the relation between hierarchical control and the optimal size of a **firm**. In this section we will give a description of Williamson's model, a model that is used as reference model by many authors, and discuss where we have to extend it in order to describe labour flows and personnel management.

Williamson's model describes a profit-maximising, price-taking **firm** with m administrative layers. Each employee can supervise s subordinates; the span of control is assumed to be the same at every layer. Production takes place only at the lowest layer, employees at higher levels spend their time with supervising the employees at the next lower level. Each production worker has the same labour productivity q ; however, due to "loss of control", productivity decreases by a constant fraction $(1-a)$ for each layer. These four parameters define the total output $Q=q(as)^{m-1}$. The costs that are involved with the production of Q consist of wages and non-labour costs. Each production worker receives a wage w ; Williamson assumes that the wage at each level is a multiple of the wage at the next lower level. Non-labour costs are assumed to be a fixed fraction of the production. Given this specification of output and costs, the profit maximisation procedure results in an optimal size of the **firm** by the determination of the optimal number of layers m .

Under reasonable conditions for the parameters, Williamson obtains several conclusions from his model. The optimal number of layers (m) increases when the degree of compliance (a) increases and when the span of control (s) increases. The optimal number of layers decreases when the wage difference between levels increases, and also when the wages increase relatively to the output price and the non-labour costs. If there is no loss of control ($a=1$), then the optimal size of the firm consists of infinitely many layers; the existence of a loss of control results in the existence of a limit on the size of the **firm**.

In a more general framework, Calvo & Wellisz (1978) show that loss of control in itself is not sufficient for the existence of a limit on the size of the firm. When employees cannot identify the moments at which they are monitored, there is no limit on the firm size. An optimal size will exist when it is impossible (e.g. due to technological reasons) to set up a

supervision system that forces employees to work 100% of the time they are being supervised. In the model of Williamson, this is guaranteed by the fixed, strictly positive control loss fraction. Furthermore, Calvo & Wellisz show that in Williamson's set-up it is not optimal to set the same wage for two successive layers. This supports Williamson's assumption of higher wages at each higher level. The literature on hierarchical models illustrates that the assumptions which ensure a limit to the optimal size of the firm under the condition of price taking and hence of perfect competition, are essential for the working of the model. We will pay special attention to this issue in the construction of our simulation model and show how the optimal size of the firm may differ when changing the parameter values with respect to these assumptions. An alternative would be to release the assumption of perfect competition and to specify a downward sloping demand curve for the product of the firm.

In a follow up paper, Calvo & Wellisz (1979) derive several propositions that are useful in building a simulation model of internal labour flows. First, they show that the choice of an optimal policy at each level in the hierarchy is independent of the optimisation at other levels. Second, it is optimal to assign employees with better qualities to jobs that are higher on the hierarchical ladder, even when jobs at every level are equally difficult. Third, it is shown that the optimal span of control is higher, the higher the rank of a job.¹ Finally, Calvo & Wellisz show that it is optimal to pay higher wages to higher-ranked employees. The optimal wage differentials between layers is greater than the differentials in effective labour per worker, i.e. the number of hours worked corrected for shirking and loss of control.

An alternative explanation for the wage differentials between hierarchical levels is based on the marginal product of employees. In the traditional view, agents with better abilities are placed in higher-ranked jobs and generate a "downward externality" for the productivity of workers lower ranks (Bhattacharya & Guasch 1988). Also tournament models give a similar wage/productivity distribution. See e.g. Lazear & Rosen (1981), who show that wages based upon the rank in the hierarchy can induce the same efficient allocation as wages based on individual output levels.

Qian (1994) shows, using a model that is essentially the same as the model of Calvo & Wellisz (1979), that their conclusion of decreasing wages and span of control throughout the hierarchy is caused by the fact that they fix the number of layers and allow the number of production workers to vary. When these two are interchanged (hence a variable number of layers but a fixed number of production workers), the result is a wage and a span of control that is constant throughout the hierarchy. Profit is unlimited when both the number of layers and the number of production workers are flexible. This statement is important for our model in which we want to keep both factors free. Underlying Qian's results is the choice of the effort levels: employees are working fully efficient, or they shirk. In case of a continuous choice of the optimal effort level, he shows decreasing wage and effort levels at lower hierarchical levels. The result on the span of control however is ambiguous.

¹ However, Keren & Levhari (1979) develop a model and conclude the opposite. They find that the optimal span of control is higher in lower-ranked jobs.

The model of Williamson is unfit for the description of worker flows through the **firm**, because most determinants which may explain the existence of internal worker flows are fixed in the model. First, the span of control is the same on each level, for each job and for each worker. Second, for workers on the production level, there exists equality in labour productivity. Hence, all workers are equal and there is only a difference between production workers and supervisors. Qualities of individuals do not play any role in the model. Furthermore, other parameters like the “control loss parameter”, the wage for production workers, and the factor that describes the difference in wages between different levels, are equal for each job. In short, there is no distinction between individuals in this model. Some of the restrictions are loosened in Calvo & Wellisz (1979), who bring in heterogeneous workers. However, a major drawback for making labour flows endogenous remains the fact that it is a static model. Before we can study the flows of workers through a hierarchically structured **firm**, a time factor has to be added and we have to introduce adjustment costs in the model. If we simply create a series of Williamson-type models over time, then there do not exist factors that prevents the firm from instantaneous changes in the number of hierarchical levels, employees and so on, because costs that are involved in the adjustment do not occur. However, in practice, it will be costly to change the hierarchical system dramatically, and even small changes will not be free of costs. We note that these adjustment and search costs are major elements in theoretical models of the flow approach to labour markets.

A very neat but difficult way to introduce the time factor and adjustment costs in the model is the development of a dynamic optimisation model. In that case, expectations of all parties should be modelled, and the optimal distribution of employees (and their qualities) over the jobs must be determined. Generally, the optimal solution of a dynamic optimisation problem is based on an initial state condition, for example the current distribution of workers over jobs, and eventually on a **final** state. It is hard to allow for unexpected quits of employees in such a framework and solve a stochastic dynamic optimisation problem. It seems possible to introduce some conditions on intermediate states, in order to keep the optimisation model manageable, but the main consequence of this kind of requirements is that the dynamic optimisation problem is broken down in a series of ‘short term’ problems, where each problem has to satisfy its own final condition. The result of each problem is the initial condition for the optimisation problem in the next period. However, in doing so we have a dynamic optimisation problem that is similar to the dynamic model that we referred to as the simplest solution to introduce the time dimension in the model: a series of static models.

The hierarchical models of Williamson and Calvo & Wellisz can be solved analytically without much problems, and it may be feasible to develop a dynamic model in this vein that can be solved analytically as well. This is true for models which do not describe the individual characteristics and careers of employees. However, in our case, where we have to keep track of each individual, it seems utterly impossible to construct a model which can be solved analytically, even when we consider a series of static models **only**. For that reason we will restrict ourselves to building a model which can only be solved numerically

by means of simulations. In this numerical process, we adopt the kind of myopic behaviour of constructing a series of static models in order to keep our optimisation problem manageable.

4. The simulation model

As we discussed in the previous section, we will not make an attempt to develop a model that can be solved analytically. We want to construct a model in which the career paths of heterogeneous individual employees can be followed. This section discusses the set-up of the model. The calibration procedure and simulation results for the basic version of the model will be presented in the next section.

The central assumption in our hierarchical model is that each employee takes the decision on whether he or she spends his/her time on the production of output or on supervising subordinates (or a combination of both) independently from the others in the firm.² We may consider this decision for each individual to be taken by the management of the firm instead of by the employees themselves. Starting with the highest-ranked person in the firm, she takes a decision on the number of subordinates that is optimal in her circumstances, by maximisation of the function

$$prf_{it} = \sum_{j=0}^{n_i} \{a(n_i)q_{j,i+1,t} - w_{j,i+1,t} - fp_{j,i+1,t}\} - AC(n_i, n_{i-1}^*), \text{ with respect to } n_i, \text{ the (potential)}$$

number of subordinates.³ In this function, the index i indicates the hierarchical level of the supervisor ($i=1$ for the top level), while the subordinates (who are identified by index j) are employed at level $i+1$. The index t indicates time; in our simulations we will assume that time is measured in years. Two parts can be distinguished in the profit function. The first part, formed by the terms under the summation sign, describes the production ($q_{j,i+1,t}$) and its direct costs: the wage ($w_{j,i+1,t}$) and the supervision costs ($fp_{j,i+1,t}$) which measure the foregone production. All three variable are measured per subordinate. The production of an individual subordinate is given by ($q_{j,i+1,t}$), but the actual production in case of co-operation with other workers can be higher or lower. We introduce a function that describes the benefits of co-operation, $a(n_i)$. It is specified as a concave parabola, implying the existence of an optimal scale of production. This specification of the production function, with increasing returns to co-operation for the first few subordinates and decreasing returns to co-operation for the latter subordinates, is the main determinant for the span of control of the supervisor. Co-operation increases production, but this effect is offset by its direct costs that are described by the function labelled ‘Supervision Costs’. It is an indicator for the value of the time that the entrepreneur has to spend on supervision of each of her subordinates; while supervising, she cannot contribute to the production. Each subordinate requires a certain amount of supervision, dependent on his qualities.

² For the sake of convenience we will assume in the remainder of the paper that the highest ranked person in the firm is female and that all others are male.

³ The production of the supervisor and her wage are not represented in the profit function because they do not affect the optimal number of subordinates.

These two functions together endogenously determine the optimal span of control. Span of control is not fixed, as it was in many earlier models.

The individual measures of productivity and costs are summed over all n_t subordinates to determine the total (potential) direct profit.⁴ The second part of the profit function, $AC(\cdot)$, describes the costs that are involved in the adjustment process. Hence it is a function of both n_t , the optimal number of subordinates, and n_{t-1}^* , the actual number of subordinates that remains from last period. Generally, a change in the number of subordinates is not free of costs since both hiring and firing are costly. Without adjustment costs it would be optimal to choose the number of subordinates that gives maximum direct profit, but the existence of costly adjustment causes a slower adjustment towards the optimal level. This is an essential feature which enables the model to describe internal and external flows of labour, and to link the model with search theory on the labour market. Moreover, the introduction of adjustment costs in the model and the fact that we want to allow for heterogeneity amongst workers makes it necessary to keep a track record of each individual worker in the simulation program. The set up of the model enables us to monitor the quality of each individual worker and to register changes over time of this quality. Information on individual qualities is used in the selection procedure when filling vacancies.

Table 1. Set-up of the simulation model

STEP 1	determination of optimal number of subordinates (per supervisor, per time period)
STEP 2	in case of vacancies: search for employees <ul style="list-style-type: none"> ▪ promotion of insiders (causes vacancy chains) ▪ hire of outsiders (training might be necessary) in case of superfluous workers: fire the least qualified subordinates (the result of this step might be that there remain unfilled vacancies)
STEP 3	repeat steps 1 and 2 for each subordinate until you reach the layer where the (optimal) number of subordinates equals zero.
STEP 4	determine the number of employees, production (optimal, actual), hiring, firing (flows, costs), organizational structure of the firm
STEP 5	<ul style="list-style-type: none"> ▪ random quits will take place ▪ increase in the experience of the employees who stay (“learning by doing”) ▪ repeat steps 1 to 4 for the next period

Maximisation of the profit function by the entrepreneur is the first step in the construction of a hierarchically structured firm. The whole simulation process can be decomposed in five steps, which are shown in table 1. After the determination of the optimal number of subordinates (step 1), the second step in the building of the firm is to be taken: if necessary, i.e. if the optimal number of subordinates n_t differs from the currently available number n_{t-1}^* , then the optimisation process will be followed by a search process, or by the firing of employees. In the latter case, where we have superfluous employees ($n_t < n_{t-1}^*$), the subordinates with the lowest qualities are fired. If such superfluous employee is located in the higher ranks of the firm, this firing process implies that the whole branch (or

⁴ The summation starts from $j=0$, what reflects the possibility that a supervisor has no subordinates.

department) of the **firm**, which apparently is not profitable, is closed down. In the former case, with $n_t > n^*_{t-1}$, there exist one or more vacant positions, for which we have to find employees who can fill them. The question is, who are available and what are the consequences when someone is hired. Here, we assume that there is a preference for internal candidates, where we select among all employees in the **firm** who are working on the next lower level (not only direct subordinates, but also employees on this level in other ‘branches’ of the hierarchical tree). If there are not enough internal candidates, or if the internal candidates fail to meet the minimum requirements, then search will be extended to external applicants. It is assumed that the qualities of external candidates follow a uniform random distribution. Two remarks have to be made. First, we assume higher minimum requirements for internal candidates because if an internal candidate is hired, the consequence is that his old position becomes vacant and a search process has to be started here as well. Second, if the qualities of an external candidate are almost sufficient, he will receive a training procedure to boost his qualities. We assume that the benefits of the training can be used in the current production, without delay. We do not consider training for incumbents. Qualities are equally important in each job, thus training would not give them any additional qualities that would make it optimal to leave their current job and move to another job inside the **firm**. However, if neither the internal candidate nor the external candidate has a sufficient quality level, the job will remain vacant, and as a consequence the actual number of subordinates is smaller than the optimal number of subordinates. If this is the case, the supervisor will spend less time on supervising and has a higher ‘residual production’.

For each filled job, i.e. for each subordinate, we will repeat the optimisation and search procedure (step 1 and 2), taking into account the central assumption that each employee takes independent decisions on whether he spends his time on the production of output, on supervising subordinates, or a combination of both. The determination of the optimal number of subordinates does not depend on the decisions that are taken at other levels and at other ‘branches’ in the hierarchy: it is a purely individual decision. The structure of the profit function has to guarantee that there is a limit on the size of **firm**, i.e. that there exists a layer for which it is not optimal to have subordinates or for which it is impossible to find applicants that meet the minimum requirements. If this is certified, the number of repetitions of step 1 and 2 is finite and it will be possible to describe the **firm**, by means of (e.g.) the number of employees, the organisational structure, the output that is generated, and by the number of unfilled vacancies (step 4 in table 1). Note that both the number of hierarchical layers as well as the span of control is determined within the model; a setting that caused troubles in Qian’s model.

After these four steps, we have established the hierarchical set up of the **firm** at the start of period t . All workers in the hierarchy stay at their job for (at least) one time period, and produce output during this period. At the end of period t (or at the beginning of period $t+1$), a random number of employees decides to quit the **firm**. Here we can think of workers finding a job elsewhere, of retirements or of workers who have other reasons for leaving the labour force. The result is the opening of vacancies at their old positions.

Furthermore, the passing of time generates an increase in the experience of those employees who are in the firm, which is depicted by an increase in their quality measure. If the growth in the qualities of an employee is insufficient to keep track of the growth of the requirements, then the employee will be fired and his job becomes vacant. Instead of immediately starting the search for candidates who can fill these vacancies (and the vacancies that remained unfilled **after** the last search process), we return to step 1, the optimisation process. There, it is determined whether it is optimal to try to find an employee for the vacancy, or whether it is better to close the vacancy without starting the search process for a new employee. It might turn out to be optimal that even more vacancies will be opened. The next steps in the table are carried out successively, as described above. In principle, it is possible to continue the process indefinitely, but we will restrict our simulations to a predetermined number of years.

Note that we do not care about the actual production and the actual costs of the **firm**. The optimisation process is directed at the expected number of employees and expected qualities of applicants, and tries to **find** out the optimal number of subordinates. The results of the search process (is it possible to **find** an employee for a vacancy, or will the position remain vacant) does not affect the optimisation process for the current employee. Of course, it affects the optimisation procedure for subsequent layers and for the next period, because it determines the starting point for these processes.

We refer to the appendix for an extensive discussion of the model specification. Here, we will only give a description of the basic structure of the profit function and the adjustment costs. Basically, the functions consist of a shift-parameter, a slope-parameter that defines the differences between hierarchical levels and a part that takes account of the qualities of the employee occupying the job.' The slope-part is structured in such a way that workers on lower levels produce and earn less than higher-ranked employees. The difference in the slope-parameters has to set a limit to the size of the firm, to guarantee that the difference between production and costs becomes so large that extra subordinates are not beneficial any more. This is a crucial feature of the model because we assume perfect competition, what implies that the price of output is not a function of the total production. Hence, we do not have a downward sloping demand curve for the product of the **firm** which would set a limit to the size of the **firm**. For simplicity we normalised the output price to one. The individual-specific part of the functions is specified to ensure that better-qualified employees produce and earn more, and need less supervision. Following our central assumption, there are no direct links between the production at different levels. The only requirement is that the position of the (direct) supervisor does not remain vacant. Production, or the quality of an employee, at level i , does not affect the production at other levels, neither does it affect the wages at other levels.

The adjustment costs $AC(n_i, n^*_{i-1})$, depend on the **number** of vacancies $n_i - n^*_{i-1}$. In the literature, quadratic adjustment cost functions -or more general strictly convex adjustment

cost functions- are most widespread, because it is possible to derive explicit labour demand functions for them. See Nickell (1986) for a discussion on shapes of cost functions. For the sake of simplicity we assume linear cost functions, which seems not to be at variance with empirical evidence as well (see Pfann and Verspagen, 1989). We assume not only asymmetry between fires and hires, but we take into account the kind of applicants that can be expected: we make a distinction between costs of search among internal candidates and search costs for external applicants. If there are vacancies, i.e. $n_t - n^*_{t-1} > 0$, we assume that the supervisor has insight in the number of subordinates that are available on the next lower level in the hierarchy, and that these employees are ready to fill the vacancy. Hence, in determining the optimal number of vacancies, it is *assumed* that the employees on the next lower level are the first group that can fill the vacancy. If the number of vacancies is higher than the number of internal candidates, the remaining vacancies are *expected* to be filled by external applicants.

5. Characteristics of the basis version of the calibrated model

Now we are to find plausible functional forms and realistic parameter values for the processes of production, coordination and search distinguished by the model. As mentioned before, there are only very few studies from which we can borrow empirical results on the labour market dynamics and on the hierarchical organisation of firms. For our calibration of quits, fires, internal and external worker flows, we base the characteristics of the benchmark representative firm of our basic simulation on a study by Hamermesh *et al.* (1996). They use data from a survey amongst firms with 10 or more employees, a stratified sample of about 2000 firms. About half of these could be used to make estimates of worker flows in 1990. These estimates are presented in table 2. It shows clearly that the majority of flows is to and from existing jobs, for inflows and outflows, as well as for internal mobility. We will consider this table as a representative firm and we calibrate upon these flows.

Table 3 presents the parameters of the model that have to be calibrated, and the values that we considered to describe best the observed flows. Before we discuss our considerations in the choice of the parameter values, first a remark on the naming of the parameters. In the functions where we have both shift as well as slope-parameters, the former ones are indicated as $\gamma_{func,1}$ while the latter are represented by $\gamma_{func,2}$, where *func* is used to indicate which function is involved.⁶

⁵ A time-varying part (trend, business cycle) is not included in the simulations, but the model is capable to handle variance over time; it is included in the model in the appendix. Moreover, some functions do not contain the slope-parameter or the quality-dependent part but only the shift-parameter.

⁶ Note that 'slope' refers to differences between the hierarchical levels, not to time differences.

Table 2. Estimates of annual worker flows in the Netherlands, 1990

Type & direction	Size
Hire to a newly-created job	3.2
Hire to an existing job	8.7
Outflow from a destroyed job	1.9
Outflow from an existing job	8.2
Internal mobility between existing jobs	1.8
Internal mobility from existing to newly-created job	0.9
Internal mobility from destroyed to an existing job	0.4
Internal mobility from destroyed to a newly-created job	0.3

Note: Hamermesh et al. (1996, table 1). The size of the flows is given in percentages of employment.

In our model, the slope-parameter in the wage function is smaller than in the production function ($\gamma_{w,2} < \gamma_{q,2}$), hence wage differentials are larger than the differences in productivity. Employees on higher levels are relatively better paid than their colleagues in the lower hierarchical ranks. This structure is in line with the models discussed in section 3, but it causes a problem in our flexible setting (see Qian, 1994). There is a risk that optimal firm size equals infinity, in case labour is too cheap in the lower ranks. A limit to the firm size has to be set by use of the parameters of the scale benefits function and the cost function.

It is reasonable to assume that the value of the slope-parameter $\gamma_{prob,2}$ in the probability density function of the quality of applicants is larger than the value in the production and wage functions: $\gamma_{prob,2} > \gamma_{q,2} > \gamma_{w,2}$. Applicants at lower levels are relatively high qualified, the actual productivity of the job decreases at a higher speed than the quality of applicants. The phenomenon of crowding-out of employees in low-quality jobs (say, lower hierarchical levels) by higher qualified personnel is another justification for this assumption.

Now we are to determine the minimum requirements for a job. According to Calvo & Wellisz (1979), a difference in requirements is not necessary to make it optimal for the firm to pay higher wages at higher hierarchical levels. However, we do not expect that a firm sets the same requirements (in terms of education, experience etc.) for managerial levels as for production workers. We assume that the decrease of quality requirements through the successive hierarchical levels emerges at a similar pace as the decrease in wages; qualities of employees must be sufficient to make them worth their earnings ($\gamma_{w,2} = \gamma_{mq,2}$).⁷

We assume that the time a supervisor has to spend on supervision does not depend on the level in the hierarchy, we set $\gamma_{fp,2} = 1$. Quality differences are defined already by the production function, and we do not expect that there is an additional difference in supervising qualities. We do account for the qualities of individual subordinates, we assume that subordinates with higher qualities require less supervision. This is an

⁷ Minimum quality requirements that decrease at a higher pace than the expected qualities of external applicants make that it is almost always beneficial to hire the applicant. Hence, minimum requirements do not help us in limiting the size of the firm.

individual effect, there is no level-effect. Note that on lower levels the productivity is smaller, so spending time on supervision is relatively more costly than at higher levels.

The next step in the calibration procedure is to set values to the various parameters of the model such that the model generates a representative firm with sizes of worker flows indicated in table 2. As far as we know, consistent data at national or **sectoral** level of wage differentials, labour productivity, quality requirements and average quality of applicants between hierarchical levels does not exist. Yet there are a few case studies for specific firms. In the calibration of our model we use data from a research project by Baker *et al.*(1993, 1994), which describes the hierarchical structure amongst management employees in a medium size American firm in the service industry, growing from 1250 employees in 1969 to 5200 in 1988. They conclude that there are eight hierarchical levels and 276 job titles, however 14 of these job titles are enough to classify 90% of the observations. The wage decline per level in their research implies a value for $\gamma_{w,2}$ of 0.85.’ The span of control in this firm ranges from 3 on the top level to 11 on the middle layer; for the three layers below this, the span of control equals one.’ However, the firm studied by Baker *et al.* is much larger than the representative firm to calibrate the basis simulation of our model. The same criticism holds for the data of a large financial institution in the UK that is used by Audas *et al.* (1997). This **firm**, observed over the period 1990-1994, has about 40000 employees. The firm comprises 13 grades with a decrease in the span of control. At the top level, span of control is about 5 employees, and this gradually decreases to values below one for the lower third of the hierarchy. Observed wage differentials imply that $\gamma_{w,2}$ should be equal to 0.75. A description of the same data but over a longer time period (1989- 1997) is given by Van **Gameren** (1998). Information on tenure and age of the employees of the **firm** can also be found there. In the choice of parameters we also had a look at this tenure patterns, but we do not formally calibrate upon them. Our model allows for differentials in the development of wages by tenure and age, but due to the rather diffuse empirical evidence on their effects (see e.g. Altonji & Shakotko (1987), Hartog *et al.* (1997), Theodossiou (1996)), we have decided not to use the parameters. The only occasion where we make use of the age of employees is the retirement decision. We have introduced mandatory retirement at the age of 60.¹⁰ This does not imply that tenure and age do not appear in our model. The promotion decision in our model is based on profit maximisation, in which the quality of the (potential) employees plays a decisive role. The quality in any period is modelled as a function of the initial quality and the tenure. Initial quality can be considered as education and experience attained before entering the **firm**, which is related with age. The further random increase can indicate additional (non-specified) education, and it depends on tenure. We constructed the quality increase function such that the highest growth in qualities is expected to occur in the first year one occupies a job, and declines quickly over the subsequent years.

⁸ Baker et al. (1994), table V.

⁹ Baker et al. (1993), table 1.

¹⁰ Mandatory retirement for most professions is at the age of 65, but in practice the number of people over 60 that is employed is very low.

Table 3. Parameter Values

	$\gamma_{func,1}$	$\gamma_{func,2}$	γ_{func}
production function	150	0.85	q
wage function	175	0.75	w
minimum requirements	250	0.75	mq
quality distribution	$\gamma_{lb,1} = 75, \gamma_{ub,1} = 225$		0.87
supervision costs	37.5	1	fp
			param.
Benefits of coop.: maximum	1.9		Max
benefits of coop.: max. reached at $n = \dots$	4.2		n_{max}
annual linear growth	0%		δ
annual growth of qualities (uniform between 0% and $\dots\%$)	20%		γ_{GRWE}
percentage of quits (at the top level)	2%		γ_{QUIT}
extra requirements for incumbent vs. external applicants	10%		γ_{RQIE}
consider training if lack of qualities smaller than $\dots\%$	20%		γ_{RQTR}
mandatory retirement age	60		γ_{RETR}
firing costs	25		$\gamma_{fr,1}$
internal search costs	33.33		$\gamma_{si,1}$
external search costs	50		$\gamma_{se,1}$

In order to achieve a **firm** size of about one hundred employees divided in four hierarchical levels, we constructed the benefits of cooperation function such that it attains its maximum for $n_{max}=4.2$ subordinates; this results in an actual span of control of (on average) five subordinates for each supervisor. In order to obtain a similar difference in earnings between the top level and the bottom level as Baker et al., we have set $\gamma_{w,2}=0.75$ in the wage equation. The other $\gamma_{func,2}$'s are based on this choice and upon the discussion above.

After the selection of the values for the $\gamma_{func,2}$'s, we now have to set values for the $\gamma_{func,1}$'s, the parameters that have no effect on the differences between hierarchical levels. We set the shift parameter in the function that describes the average quality of an external applicant lower than the corresponding parameter in the function that describes the minimum requirements, $\gamma_{prob,1} < \gamma_{mq,1}$. Due to the choice of $\gamma_{prob,2}$ and $\gamma_{mq,2}$, the implication is that for lower-ranked jobs almost every applicant has the capacities to do the job, while at the higher levels more applicants will be rejected. We selected $\gamma_{mq,1}=250$ and we define a probability density function of the qualities of external applicants that has an expected value with $\gamma_{prob,1}=150$. Furthermore, for the wage function we selected a $\gamma_{w,1}$ that is higher than $\gamma_{q,1}$, the shift-parameter of the production function, The result is that employees at the lowest level earn a wage that is approximately equal to their productivity, while the managers earn more than their productivity (this is caused by the different slopes, indicated by the parameters $\gamma_{w,2}$ and $\gamma_{q,2}$). Our choice of $\gamma_{w,1}=175$ yields wages that are around the average expected quality of an external applicant, while $\gamma_{q,1}=150$ yields the desired relationship between wages and the productivity of employees.

The calibrated search costs are based on the results of Pfann & Verspagen (1989) by a linear approximation of their quadratic adjustment cost function. The plot of their estimated cost function shows that initially, hiring costs seem to be a little higher than the firing costs, while for more expansionary firms the hiring costs increase exponentially. Because the first option to fill vacancies is through internal movements, we assume that external search is more expensive than internal search. We set the external search costs equal to 50, roughly equal to the annual salary of an employee at the lowest level. Internal search costs are set at two-thirds of the external search costs. Both are assumed to be higher than the firing costs, which we will set equal to half of the external costs. Although the costs of searching among internal candidates is smaller than costs of external search, there is another factor that plays a role here: internal mobility implies that an employee who fills a vacancy, creates a new vacancy for the job he leaves. We account for this additional factor by levying an extra requirement on internal candidates. We assume that the quality of an internal candidate has to be 10% above the normal minimum requirements to make it worthwhile to promote him. For internal candidates, we do not consider the possibility of training to increase his qualities. If an internal candidate fails the minimum requirements to fill a vacancy, it is better to keep him in his current position. On the other hand, for external candidates, training can be beneficial; the alternative is that the vacancy cannot be filled and does not generate any production at all. Hence, if the candidate does not meet the minimum, but comes close to it, training might bring him up to the minimum level. We consider training when the gap between required qualities and the actual quality is smaller than 20%. We do not provide any explicit training for employees once they have a job within the firm, but we assume that the qualities of incumbents grow every year. Learning-by-doing increases the qualities with a random percentage between 0 and 20% in the first year in the job. Each subsequent year the maximum increase is halved.

The calibrated parameters given in table 3 and discussed above give rise to a representative **firm** for which the simulation results are presented in table 4. The flows are based on simulations over a period of 50 years, replicated 100 times to account for the random processes incorporated in the model. Averages are taken over the last 45 years, because in the initial five years the **firm** has to grow to its optimal size. This process involves more **(in)flows** than in the latter phase.” Note that random procedures occur in three places in the simulation model. First, and most important, the qualities of external applicants are drawn **from** a uniform distribution.” If no suitable candidate applies for a job, then the implication is that a job will remain vacant, which has major consequences for the development of the firm. The second important random process is the occurrence of quits. Each period a percentage of the employees voluntarily leaves the firm. Thirdly, the accumulation of qualities is modelled as a random process. Once an employee is hired, he will gain qualities.

¹¹ The standard deviations are calculated for each year, based on the 100 simulations; in the table we present the average standard deviations over the last 45 years.

¹² The age of external applicants is also drawn **from** a uniform distribution. This randomization is of minor importance, at the moment it only plays a role in the retirement decision.

Table 4. Results of the simulations (percent of employment)

Type & direction	Target size	Simulations	(st.dev.)
Number of employees in the firm	136	120	2.2
Hire to a (newly)-created job	3.2	3.0	1.6
Hire to an existing job	8.7	6.8	2.3
Outflow from a destroyed job (Fires)	1.9	2.3	1.4
Outflow from an existing job (Quits)	8.2	7.5	2.3
Internal mobility to a (newly) created job	2.2	0.002	0.02
Internal mobility to an existing job (by direct subordinate)	} 1.2	0.2	0.4
Internal mobility to an existing job (from other 'branch')		1.3	1.1

Note: The 'target size' is drawn from table 2. All numbers (except the number of employees) are percentages of employment.

As can be read from table 4, the average firm size does not fluctuate much between the various simulations: the standard deviation is small. The table also shows that our calibrated model is able to generate the target values with respect to the outflow of employees, and its distribution over quits and fires, reasonably well. However, in this case the random processes which hit the firm cause much more variation than in the case of the firm size. This is shown by the fact that the standard deviation of the flows are rather high in the simulation experiments. The calibration of the inflow of workers, and especially of the internal flows appears to be the most difficult. Internal mobility towards new jobs is extremely low, which is caused by the fact that new jobs are almost exclusively created on the lowest level in the hierarchy (the firm stabilises and does not create new jobs on the high levels), implying that internal flows are excluded and external inflow is the only alternative. On the other hand, it turns out that external inflow is almost exclusively directed towards the lowest level in the hierarchy and hardly occurs at the other levels.

Internal flows on the same hierarchical level (i.e. horizontal flows) do not exist in the current model. Due to the fact that we have only one quality indicator, it is never beneficial to change departments while you remain on the same level; productivity does not differ per 'branch' while a move would involve additional (adjustment) costs. Horizontal flows could be introduced by collecting workers who are fired due to the quit or the fire of their supervisor in a pool and give them the opportunity to stay in the firm on the same (or a lower) level. Another way to implement horizontal internal mobility would be to differentiate between quality improvements: workers that make horizontal moves may be given a higher chance to obtain large quality improvements each year. However, such property will only generate horizontal internal labour flows in case the dynamic maximisation of the profit function of the firm takes place over a multiperiod horizon.

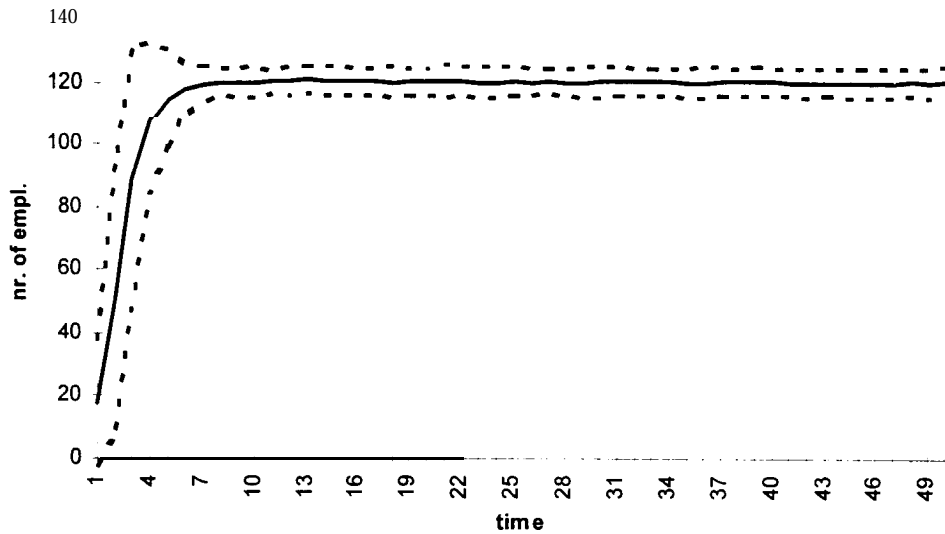


Figure 1. The number of employees and the confidence interval (2 times the stand.dev.)

Figure 1 illustrates that the total employment in the firm (averaged over 100 simulations) has stabilised within five years. Similar figures can be examined for the worker flows: flat profiles around the numbers presented in table 4, but with wider bounds than for the number of employees; at each point in time, the standard deviations of the simulated worker flows are large. The number of unfilled vacancies stabilises at about five vacancies in every period, which is about 4% of the total number of jobs. Once it has stabilised, the firm always consists of four hierarchical layers (including the top level where the entrepreneur is employed), and for each layer the span of control is about five subordinates on average (except for the last layer, where the number of subordinates by definition equals zero). Vacancies that occur after random quits at higher levels are filled by internal mobility, vacancies at the fourth (i.e. the lowest) level are filled by external applicants. The vacancies that remain unfilled at the end of a period are almost exclusively located in the fourth layer.

6. Sensitivity analysis on the specification of the model

The analysis in section 5 is based on a fixed set of parameters. Based on 100 simulations, we generate a representative firm by taking the average over the simulations. The 100 firms constructed during the simulations are homogeneous with respect to the parameters. Differences occur only due to the random procedures incorporated into the model, which generate heterogeneous employees. Therefore, the enormous heterogeneity amongst firms, which is revealed by the analysis of panel data sets in the vein of Davis and Haltiwanger, has to be implemented in the simulation model by means of parameter changes (and by changes in the functional forms of the processes and changes in assumptions, which we leave to later studies). In this section, we discuss the results of runs of simulations each

with (in each run) one of the parameters below or above its base value as given in table 3. We did this to gain insight in the sensitivity of the results to the size of the parameters, and to gain insight in the sources of heterogeneity amongst firms.

The results are less sensitive to the shift- ($\gamma_{func,1}$) parameters than to the slope- ($\gamma_{func,2}$) parameters. The latter ones **define** the differences between the hierarchical levels, and are included in the functions in an exponential manner. Therefore, a small change can cause large changes on the lower levels. The model is most sensitive for the values of $\gamma_{q,2}$ and $\gamma_{w,2}$, the slope-parameters in the production function and in the wage function; an increase or a decrease of 2% is **sufficient** to cause huge changes in the size of the firm. For these two parameters, a change of less than 2% leads to the inclusion of an extra layer, or a layer is scrapped from the hierarchy. A change of about 3% in $\gamma_{fp,2}$ has the same large effect. The only other parameter for which a difference of 5% causes one layer more (or less) is the parameter that defines the maximum benefit of co-operation (*max*). All the other parameters need larger shifts to have a noticeable effect the size of the firm. Even the shift-parameters in the production and wage functions show only small effects. A parameter that does not result in a different number of layers but has an effect on the size of the firm is n_{max} , the number of subordinates which gives the maximum benefits of cooperation. A 5% increase in its value results in a clear increase in the size of the firm - from 120 to 160: the number of subordinates increases.

Internal mobility is not very sensitive to changes in parameters. A substantive change in the internal flows towards existing jobs occurs only in the case of an addition or a deletion of a layer to or from the hierarchy. Internal flows towards newly created jobs occur rarely, whatever the parameter values are. When the number of layers changes, this clearly works out on the size of internal flows towards existing jobs: larger firms show more internal flows. That the outflow through quits hardly changes when the size of the firm remains constant is in line with the expectations. Quits occur randomly, with an increasing probability for employees in the lower-ranked layers, so a major change in the number of quits occurs only when the number of hierarchical layers changes. A higher quit probability leads to somewhat higher flows, both external and internal. Outflow through fires (job destruction) is fairly constant. Except the parameters that change the size of the firm only the average quality of applicants, the minimum requirements and the quit probability do affect the outflows. The same parameters affect the external inflows.

Summarising, the slope-parameters (the $\gamma_{func,2}$'s) in the production and wage functions are the main determinants for the number of layers in the firm, and, therefore, also for the number of employees. The other main determinants of the number of employees is the slope parameter of the supervision costs and the function that **defines** the maximum benefits of cooperation. The **shift** factors in the production and wage function ($\gamma_{q,1}$ and $\gamma_{w,1}$) need a change of about 8% to cause one layer more or less. n_{max} has an effect on the number of employees but not on the number of layers. These parameters also have considerable effects on the external flows to and from the firm. External flows are also sensitive to the slope-parameter in the functions describing the minimum requirements and

the quality distribution ($\gamma_{mq,2}$ and $\gamma_{prob,2}$), while the shift-parameters of these functions ($\gamma_{mq,1}$ and $\gamma_{prob,1}$) mainly affect the outflow from destroyed jobs (fires). All other parameters, including those describing the cost function, have only minor effects on firm size and on external flows. Internal flows show hardly any sensitivity to whatever parameter change, unless the size of the firm changes dramatically.

7. Conclusions and scope for future research

The simulation model of this paper provides a first attempt to combine elements of search theory in explaining labour flows with the analysis of the optimal hierarchical structure and size of a firm from the literature on industrial organisation and internal labour markets. In order to let the model generate internal and external labour flows, the hierarchical models of Williamson (1967) and Calvo & Wellisz (1978, 1979) are extended with a number of features. Firstly and most importantly we allow for heterogeneous labour, which requires that we have to record the quality and position in the firm of each individual worker in the course of time. Moreover we include various costs associated with labour market dynamics, such as search costs, hiring, firing and training costs, which are derived from the assumptions and empirical findings on these costs in theories of labour search and labour demand. Finally we confront the firm described by our model with random shocks with respect to quits, the quality of applicants and quality improvements of the incumbent personnel. These stochastic processes are a major source for initiating labour flows within the firm.

Of course, there is a price for these additions to the neat hierarchical models of the internal organisation literature. We have to make a number of additional assumptions for which we could not find a sound empirical foundation. Moreover, the model is no longer analytically tractable, but we have to resort to simulation exercises in order to illustrate the working of the model. Therefore we have to make decisions on the functional forms which describe the major processes incorporated into the model and we have to set plausible values to the parameters. In this calibration procedure we tried to exploit as much information as possible from the existing literature. In doing so we aimed that the basic version of the model generated the average sizes of firms and labour flows found by Hamermesh *et al.* (1996) in a panel data analysis for the Netherlands. The parameter values with respect to wage differentials, hierarchical structure and span of control of supervisors are derived from a study by Baker *et al.* (1993, 1994). Finally, the search cost included in our calibration are based on the results from an empirical study by Pfann & Verspagen (1989) for the Dutch manufacturing sector.

In spite of the fact that the model still gives a very stylised representation of labour dynamics and personnel decisions within the firm, it already provides some important insights into the organisational dynamics of the firm. A sensitivity analysis of the parameters of the various processes reveals that the major source of heterogeneity amongst firms results from the assumptions on the production and wage structure. These functions

are the major determinants of the number of hierarchical levels in the firm, hence cause the largest fluctuations in the size of the firms.

In this paper we focused on the working of the model and the calibration procedure. It gives a general impression of the mechanism at work in the personnel management of firms. However, extensions of the basic simulation model that is presented enable us to review a number of specific questions. Examples of these questions are:

1. Under what conditions are internal labour market flows procyclical, and under what conditions anticyclical?
2. How does wage flexibility affect internal and external labour market flows?
3. How does an increase in the variance of the firm specific demand shocks (idiosyncratic shocks) affect internal and external labour market flows?
4. How is the ratio between labour turnover and job turnover (length of the vacancy chain) affected by demand shocks, by supply shocks and by idiosyncratic shocks?

The questions above will be investigated in a follow up to this paper. Before we review those questions, we will build new elements into the model, and allow for more flexibility on the major assumptions. For instance, we will extend the model so that it allows for horizontal mobility within the firm as well. Up to now the model only describes vertical mobility. A further possible modification in the working of the model is to alter the perfect competition assumption on the product market and let the firm be faced with a downward sloping demand curve.

Such calibrated hierarchical models of the firm can also be used for the analysis of labour market dynamics at the macro level. Simulation exercises may reveal the dynamic reactions of the firms to exogenous shocks with respect to, for instance, a higher average quality of applicants as a result of more public expenditure on education, a change in hiring and/or firing costs, a change in aggregate demand, a change in the probability of quits due to cyclical movements, etc. Such analysis of the propagation of shocks at the macro level could take account of the heterogeneity of firms by a weighted average of simulation result for models of firms with different parameter values, which mimic the actual composition of the various types of firms in the economy.

All changes in the design of the model can be made in a rather arbitrary fashion by casual inspection of the way firms operate in the real world. Apart from plausibility, the only condition when incorporating these changes is that in calibration the model generates labour flows which are in accordance with the scant empirical literature in this field. Therefore, a major task for future research is to collect further data on the organisational structure and dynamics of personnel within firms. The model building exercise and simulation experiments of this paper reveal which kind of data are important in this respect.

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Appendix. Specification of the model

In every period t , each supervisor (occupied at hierarchical level i) determines his optimal number of subordinates n , by maximisation of his ‘individual’ profit function prf_{it} with respect to the number of subordinates.¹³ In the optimisation process, he (for the highest rank, $i=0$: she) takes into account the structure of production, wages, other costs, the (expected) qualities of applicants, and the currently available number of subordinates, n^*_{i-1} . By assumption, $n^*_0=0$, hence we start to build a firm ‘from scratch’ in period $t=1$.

The profit function is specified as

$$prf_{it} = \sum_{j=0}^{n_i} \{p_t a(n_i) q_{j,i+1,t} - w_{j,i+1,t} - fp_{j,i+1,t}\} - AC(n_i, n^*_{i-1}),$$

while its constituent parts are the following functions:

Production	$q_{j,i,t} = \gamma_{q,1} (\gamma_{q,2})^i g(t) c_{j,i,t}$
Wage	$w_{j,i,t} = \gamma_{w,1} (\gamma_{w,2})^i (\gamma_{w,3})^{tenF_{jt}} (\gamma_{w,4})^{tenJ_{jt}} (\gamma_{w,5})^{Age_{jt}} A_t c_{j,i,t}$
Supervision Costs	$fp_{j,i,t} = \gamma_{fp,1} (\gamma_{fp,2})^i g(t) (1/c_{j,i,t})$
Benefits of Cooperation	$a(n) = a n^* + b n + 1,$
(If $j=0$, i.e. there are no subordinates, then	$q_{0,i,t} = w_{0,i,t} = fp_{0,i,t} = 0.)$

The variables $tenF_{jt}$, $tenJ_{jt}$, and Age_{jt} , indicate the tenure of employee j with the firm, the time he is employed in his current job and his age. A_t allows for autonomous wage increases, $A_t = 1 + \gamma_{w,6} t$. This describes the wage effect independent of the effect of increases in quality. Productivity increases are only regulated through the quality measure.

The function p_t defines the price of the output as it will be received by the firm. Currently, it is purely exogenous and depends only on time, $p_t = 1 + \delta t$. It can be extended and used to describe business cycles, but in the current settings we have chosen for $\delta=0$.

The function $g(t)$ defines a time trend, $g(t) = 1 + \mu t$. This trend function is not used in the current simulations; in the model applied in this paper we have $\mu=0$. Probably it will be thrown out.

The relative quality measure c_{jt} is defined as

$$c_{jt} = \frac{qual_{jt}}{E(qA_{it})} \quad \begin{array}{l} \text{if the job is occupied by subordinate } j, \text{ and} \\ = 1 \quad \text{if the job is vacant,} \end{array}$$

with $qual_{jt}$ the actual quality of subordinate j , and $E(qA_{it})$ the expected quality of an external applicant.

Dynamics are incorporated in the model through the adjustment costs,

$$AC(n_i, n^*_{i-1}) = \begin{array}{ll} fr_{it} | n_i - n^*_{i-1}|, & \text{if } n_i - n^*_{i-1} < 0, \\ si_{it} | n_i - n^*_{i-1}|, & \text{if } 0 < n_i - n^*_{i-1} \leq n^{int}, \\ se_{it}, n_i - n^*_{i-1}|, & \text{if } n_i - n^*_{i-1} > n^{int}, \end{array}$$

with n^{int} the number of potential internal candidates for the job (defined as: the number of employees on the next lower level). The three constituent parts of the adjustment cost function are:

firing costs	$fr_{it} = \gamma_{fr,1} g(t),$
internal search costs	$si_{it} = \gamma_{si,1} g(t), \text{ and}$
external search costs	$se_{it} = \gamma_{se,1} g(t)$

The formulas above govern the optimisation procedure, the first step in the creation of the firm. The second step consists of the search for applicants. It is assumed that the qualities of applicants can be observed without error. Internal candidates are the employees who are employed on the next lower level in the hierarchy, not only direct subordinates but also employees at this level in other ‘branches’ of the hierarchy. If internal candidates are not available or miss the required capacities, then external candidates come in the picture. We assume that for each vacancy exactly one external applicant arrives, who is fully described by his quality measure, which is drawn from a uniform probability density function with upper and lower bounds that vary per level:

¹³ We leave out the index for the supervisor. It would show up only in the prf and in the number of subordinates n , but does not play any role in the analysis.

$$qA_{it} \sim \text{UNIF}(lb_{it}, ub_{it}),$$

$$lb_{it} = \gamma_{lb,1} (\gamma_{prob,2})^t g(t),$$

$$ub_{it} = \gamma_{ub,1} (\gamma_{prob,2})^t g(t),$$

hence the expected quality is equal to $E(qA_{it}) = \gamma_{prob,1} (\gamma_{prob,2})^t g(t)$, with $\gamma_{prob,1} = (\gamma_{lb,1} + \gamma_{ub,1})/2$.

The minimum requirements for an applicant are given by $mq_{it} = \gamma_{mq,1} (\gamma_{mq,2})^t g(t)$, but internal and external candidates are treated in a different way. An internal applicant is only hired if his qualities are clearly better than the ‘basic’ requirements, the minimum requirements for an internal applicant are given by $mq_{i,t}^{int} = (1+\gamma_{RQIE}) mq_{i,t}$, with $\gamma_{RQIE} > 0$. For an external applicant, the requirements are lower than the ‘basic’ requirements; training will be provided when the applicant almost meets this basic level: if $qA_{it}/mq_{i,t} > (1-\gamma_{RQTR})$, with $\gamma_{RQTR} > 0$, the applicant will be hired.

The optimisation and search process (steps 1 and 2) are fully captured now. Steps 3 and 4 do not require any new equations, but in step 5, the progress to the next time period, two random processes are going on. First, with every employee we associate a quit probability, or more precise, a non-quit probability: the probability that an employee at level i does not quit equals $(1-\gamma_{QUIT})^i$. However, there is a retirement age. Once an employee reaches the age given by γ_{RETR} , he will be retired. This is counted as a quit. Second, for each worker who is employed in the firm (and who does not quit), we assume that there is a tenure effect that leads to the accumulation of quality. The effect depends on the tenure in the current job, and has a random component: the qualities for each individual in the next period are given by $qual_{j,t+1} = qual_{j,t} (1+2^{1/2})^{\gamma_{ten,j,t}} U$, where U is the random factor, drawn from a uniform distribution, $U \sim \text{UNIF}(0, \gamma_{GRWE})$.